* 1

(a)

(i) 3 (1,2);

P\_1 : (0, 9/5) z = -3.6, stop, can’t improve

P\_2 : (1, 2) z = -3, stop, optimal

(ii) No, negative coeffs.

(iii) Combines cuts with BnB (see last slide of BnB)

(b)

Let x\_K be 1 when investment K is taken, 0 otherwise

p\_K be profit for investment K, c\_K be capital for investment K

Max sum of x\_K p\_K over all K in [A,G]

x\_A + x\_B <= 1

x\_C + x\_D <= 1

x\_C <= x\_A + x\_B

x\_D <= x\_A + x\_B // NB potentially could combine last two lines into x\_C + x\_D <= x\_A + x\_B but this isn’t a “nice” constraint as it depends on the other constraints? Idk

sum of all x\_K c\_K <= 100 for all K in [A,G]

x\_K in {0, 1} for all K in [A, G]

2

(a)

(i) 19 ⅓ (4 ⅔, ⅔)

(ii) Further pivot to remove it from the basis

(iii) min 6y\_1 + 3y\_2 + 10y\_3

y\_1 + y\_2 + 2y\_3 >= 4

2y\_1 – y\_2 + y\_3 >= 1

y\_1 free, y\_2 <= 0, y\_3 >= 0

(iv) see slides, Duality(11/46). A tip: memorise the shape (Ax)^T y, everything comes easily after.

(b) See tutorial & case study 6. Basically the optimal at relaxed have all integer solns. Definition: A sq matrix with all sq submatrics having determinant -1, 0, 1; more strictly the 0 can be restated, giving

**A square matrix with all non-singular sq submatrixes having determinant -1 or 1.**

Integer programming can be shown to be NP-hard (intractable). LP relaxation is polynomial time. So totally unimodular matrices, which have LP relaxation’s optimal soln integral, are polynomial time.

3

(a) (i) <OUT OF SYLLABUS>

(b) See tutorial; for (ii) just remove the Wed line

4 (a) (i)

The two numbers are how many divisions placed at each mountain pass e.g. 21 means 2 div in pass A, 1 div in pass B

|  |  |  |  |
| --- | --- | --- | --- |
|  | **02** | **11** | **20** |
| **03** | 1 | -1 | -1 |
| **12** | 1 | 1 | -1 |
| **21** | -1 | 1 | 1 |
| **30** | -1 | -1 | 1 |

Regarding dominated strategies, for Colonel Rogers, it is not really a good idea to put all 3 divisions in either of the two passes so he has a better chance of winning if he chooses either 12 or 21. Once the enemy realize that Colonel will never follow 03 or 30 he is much better off choosing either 20 or 02 since choosing is never going to beat 21 or 12.

(ii) max min p\_i q\_j a\_{ij} for all i in [1,4] and j in [1,3]

max t

s.t. sum p\_i a\_{ij} over all i in [1,4], for all j in [1,3]

(iii) see slides. In short V\_{CP} = V\_{RP} = V because of strong duality.

(b)

(i) 1/5 x\_3 + 3/5 x\_4 – x\_5 +

\xi = 3/5

(ii) Tableau should be easy.   
  
When all BV’s are integers at optimal, or if there doesn’t exist any more legal cuts, i.e. y\_{i0} = int for all i in I. The former means terminate with optimality (success), the latter means infeasible.

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